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Negative Mass Propulsion

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Summary

It is easy to prove that there are negative masses all around us, albeit hidden behind positive masses. But their use for propulsion by reducing the inertia of matter, for example in the limit of macroscopic bodies with zero rest mass, depends on a technical solution to free them from their imprisonment by positive masses. It appears that there are basically two ways this might be achieved: 1. By the application of strong electromagnetic or gravitational fields or by high particle energies; 2. By searching for places in the universe where nature has already done this separation, and from which the negative masses can be mined.

The first of these two possibilities is for all practical means excluded, because if possible at all, it would depend on electromagnetic or gravitational fields with strengths beyond what is technically attainable, or on extremely large particle energies likewise not attainable.

With regard to the 2nd possibility, it has been observed that non-baryonic cold dark matter tends to accumulate near the center of galaxies, or places in the universe which have a large gravitational potential well. Because of the equivalence principle of general relativity, the attraction towards the center of a gravitational potential well, produced by a positive mass, is for negative masses the same as for positive masses. Large amounts of negative masses might have over billions of years been trapped in these gravitational potential wells.

Now it just happens that the center of the moon is a potential well, not too deep that it cannot be reached by making a tunnel through the moon, not possible for the deeper potential well of the earth, where the temperature and pressure are too high. Making a tunnel through the moon, provided there is a good supply of negative mass, could revolutionize interstellar space flight. A sequence of thermonuclear shape charges would be required to make such a tunnel technically feasible.

1. Introduction

If we extend the law of gravity to negative masses, but hold onto the equivalence of inertial and gravitational masses, we have to distinguish between the following four cases, if a test particle is placed near a gravitational field producing mass (Table 1):

Table 1. Interactions

Case	Gravitational field producing mass	Mass of test particle	Motion of test particle
1	+	+	attraction
2	+	–	attraction
3	–	+	repulsion
4	–	–	repulsion

Under the principle of equivalence if a negative test mass particle would be placed in the gravitational field of earth, it would not fall upwards, as happens in science-fiction antigravity machines. A test particle, regardless of whether it has positive or negative mass, would there always fall down. It would fall upwards only if placed in the field of a large negative mass.

A somewhat different situation arises if both masses, the field producing mass and the mass of the test particle, have the same absolute value but are permitted to have different signs. There we have to distinguish between the cases shown in Figure 1.

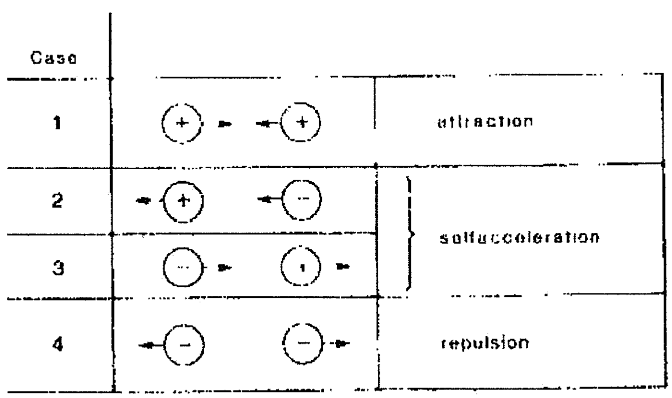

Figure 1. Forces

If both masses are positive, we have the usual Newtonian attraction. For negative masses, the force has the same magnitude but is repulsive. A quite different situation exists if one mass is positive and the other one is negative. With both forming a mass dipole, the system becomes self-accelerating, because one mass is repelled and the other one attracted. With the two masses having opposite sign, the total energy and momentum of the combined system remains zero for all times, leaving intact the conservation laws of energy and momentum. Under its self-acceleration, the mass dipole would eventually reach the velocity of light. It is this property of self-acceleration without expenditure of energy that has intrigued many researchers and raised the prospect of a propulsion system without limits. We remark that even without an appreciable gravitational interaction, a mass dipole with zero, or close to zero inertial mass, could be accelerated to very high velocities with negligible jet power and energy.

No matter how strange the properties associated with negative masses appear to be, there can be little doubt that they can be incorporated into Einstein's gravitational field theory as long as they do not violate the principle of equivalence. In particular, the well known Schwarzschild solution for a positive mass *M*

$$ds^2 = \frac{dr^2}{1-2\gamma M/c^2r} + r^2(d\theta^2 + \sin^2\theta d\varphi^2) - (1-2\gamma M/c^2r)c^2dt^2 \qquad (1)$$

can be extended to a negative mass, simply by replacing *M* with –*M*:

$$ds^2 = \frac{dr^2}{1+2\gamma M/c^2r} + r^2(d\theta^2 + \sin^2\theta d\varphi^2) - (1+2\gamma M/c^2r)c^2dt^2, \qquad (2)$$

where γ is Newton's constant.

One therefore has to raise the question if nature has not made use of negative masses somewhere. Over and over again we have found that what is possible, within the framework of the fundamental laws of physics, exists. Only one important physical set of laws, Einstein's special theory of relativity appears to forbid the existence of negative masses. This is because in a relativistic quantum field theory the particle number is not a conserved quantity, and the existence of negative masses would make all matter unstable against decay into negative masses.

Apart from Einstein's purely kinematic interpretation of special relativity, being the expression of a Minkowskian space-time structure, there is an older alternative dynamic interpretation by Lorentz and Poincaré. In it space and time are absolute, but it can explain all relativistic effects as well. It assumes the existence of an aether, with all objects in absolute motion through the aether suffering a Lorentz contraction and time dilation. If this aether has a grainy structure, characterized by some smallest length (e.g., the Planck length ~ 10^{-33} cm), then according to Heisenberg's uncertainty principle special relativity would ultimately break down at a high energy. If the length is very small, this energy can be so high as to be far beyond the capabilities of any existing particle accelerator or even beyond the high energy of cosmic ray particles, making both interpretations of special relativity experimentally indistinguishable at the energies presently available.

2. The Theory of Bondi

The first attempt to introduce negative masses into general relativity to describe a mass dipole was made by H. Bondi [1]. For a uniformly accelerating mass dipole Bondi uses the axially symmetric metric by Weyl and Levi-Civita [2]:

$$ds^2 = e^{2\varphi} dt^2 [e^{2\sigma}(dr^2 + dz^2) + r^2 d\theta^2], \tag{3}$$

where $\varphi = \varphi(r,z)$ and $\sigma = \sigma(r,z)$ satisfy

$$\left(\frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{\partial^2}{\partial z^2} \right)\varphi = 0 \tag{4}$$

$$\frac{\partial \sigma}{\partial r} = r\left[\left(\frac{\partial \varphi}{\partial r} \right)^2 - \left(\frac{\partial \varphi}{\partial z} \right)^2 \right] \tag{5}$$

$$\frac{\partial \sigma}{\partial z} = 2r \frac{\partial \varphi}{\partial r}\frac{\partial \varphi}{\partial z}\,. \tag{6}$$

Inserting (3-6) into Einstein's nonlinear gravitational field equations, one obtains four nonlinear partial differential equations ($\kappa = 8\pi \gamma / c^4$) given by

$$-\kappa\rho = -\kappa T_0^0 = e^{2(\varphi-\sigma)}\left[-2\nabla^2\varphi + \nabla^2\sigma - \frac{1}{r}\frac{\partial\sigma}{\partial\tilde{\rho}} + \left(\frac{\partial\varphi}{\partial\tilde{\rho}}\right)^2 + \left(\frac{\partial\varphi}{\partial\tilde{z}}\right)^2\right] \tag{7}$$

$$-\kappa p_{11} = -\kappa T_1^1 = \kappa T_2^2 = \kappa p_{22} = e^{2(\varphi-\sigma)}\left[\frac{1}{r}\frac{\partial\sigma}{\partial\tilde{\rho}} - \left(\frac{\partial\varphi}{\partial\tilde{\rho}}\right)^2 + \left(\frac{\partial\varphi}{\partial\tilde{z}}\right)^2\right] \tag{8}$$

$$-\kappa p_{33} = -\kappa T_3^3 = e^{2(\varphi-\sigma)}\left[\nabla^2\sigma - \frac{1}{r}\frac{\partial\sigma}{\partial\tilde{\rho}} + \left(\frac{\partial\varphi}{\partial\tilde{\rho}}\right)^2 + \left(\frac{\partial\varphi}{\partial\tilde{z}}\right)^2\right] \tag{9}$$

$$-\kappa T_{12} = 2\frac{\partial\varphi}{\partial\tilde{\rho}}\frac{\partial\varphi}{\partial\tilde{z}} - r\frac{\partial\sigma}{\partial\tilde{z}}. \tag{10}$$

In solving these equations Bondi assumes that φ is small, which then also implies that because of (5) and (6) σ is small by the second order, reducing the solution of the problem to the linear Laplace equation of the scalar Newtonian potential in empty space. Making this assumption, Bondi can reproduce the uniform acceleration of the mass dipole, as it is expected from an elementary analysis. It is here that we must disagree with Bondi[1], because it can be shown that the nonlinearity of the gravitational field equation leads to a very different result. The nonlinearity also sheds light on why it is so difficult to separate negative from positive masses, whereby negative masses are all around us, but imprisoned by positive masses.

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[1] The author had the pleasure to meet Prof. Bondi on a common flight from Graz, Austria in 1993 (we both are members of an academy which had a meeting in that year in Graz), and ask him how his solution can be correct since it does not include the field of the positive gravitational field mass of a mass dipole. This problem will be analyzed in the next section, and its solution has far reaching consequences.

3. Hund's Nonlinear Newtonian Theory of Gravity

As explained by Hund [3], already Newton's theory of gravity, in conjunction with the postulates of special relativity, leads to a nonlinear theory of gravity. With this model theory of gravity the nonlinearity of the gravitational field can be much better explained than with Einstein's theory.

Hund begins with the force ***F*** acting on a mass *m* in a "merry-go-round":

$$\mathbf{F} = m\mathbf{g} + \frac{m\mathbf{v}}{c} \times \mathbf{G} \,. \tag{11}$$

If $\mathbf{g}$ is the gravitational acceleration by real masses with the density ρ, one has in Newton's theory

$$div\,\mathbf{g} = -4\pi\gamma\rho \,. \tag{12}$$

In the merry-go-round $\mathbf{g}$ has a vertical component from the earth's gravitational field, but also a radial component by the radial centrifugal acceleration which is not source-free. With $|g| = \omega^2 r$, where $\omega = 2\pi/T$, with *T* the time of revolution, and *r* the radial distance from the center of the merry-go-round, one has

$$div\,\mathbf{g} = 2\omega^2 \,. \tag{13}$$

Comparing (13) with (12) one sees that the centrifugal force corresponds to a gravitational repulsion of a homogeneous mass density

$$\mu = -\frac{\omega^2}{2\pi\gamma} \,. \tag{14}$$

For a typical "merry-go-round" one has *T* = 10 sec, and hence, $\omega = 0.6 s^{-1}$. For this example one obtains from (14), $\mu = -10^6\, g/cm^3$, taken as an absolute value about equal to the mass density of a white dwarf.

The mass density (14) is not fictitious but represents physical reality. According to Einstein's $E = mc^2$ one obtains for an electric field $\mathbf{E}$ the mass density

$$u = \frac{\varepsilon_0 |\mathbf{E}|^2}{8\pi c^2} > 0 . \tag{15}$$

Replacing ε_0 with $-(1/\gamma)$, one obtains the energy density of the gravitational field

$$u = -\frac{g^2}{8\pi\gamma} < 0 . \tag{16}$$

It possesses the (negative) mass density

$$\mu = -\frac{g^2}{8\pi\gamma c^2} . \tag{17}$$

Besides the field g, we have on a "merry-go-round" the Coriolis force field

$$\mathbf{G} = 2c\boldsymbol{\omega} . \tag{18}$$

By setting $G = g$ and inserting G into (17) with $g^2 = G^2$, one obtains the mass density (14). This means the centrifugal force is the gravitational force associated with the mass density of the Coriolis field.

But where is this huge negative mass coming from? The obvious answer is by the very large vacuum energy, making itself felt by going to an accelerated frame of reference.

In Mach's principle the motion of the distant galaxies as seen in an accelerated frame of reference is responsible for the inertial forces. But this idea is wrong because if by some miracle the distant galaxies were to be set into motion, it would take a long time before their fields propagating with the velocity of light would reach the earth.

Adding the mass of the Coriolis field to the right side of the equation (12), we have by putting it on the left hand side

$$div\,\mathbf{F} - \frac{1}{2c^2}\mathbf{G}^2 = -4\pi\gamma\rho . \tag{19}$$

In one further step one should have

$$div\,\mathbf{F} - \frac{1}{2c^2}(\mathbf{F}^2 + \mathbf{G}^2) = -4\pi\gamma\rho \qquad (20)$$

or if $\mathbf{G} = 0$ and $\mathbf{F} = -\nabla\phi$, where ϕ is the Newtonian gravitational potential, one obtains for Poisson's equation:

$$\nabla^2\phi = 4\pi\gamma\rho - \frac{1}{2c^2}(\nabla\phi)^2\,. \qquad (21)$$

According to (21) the positive mass as the source of the Newtonian potential is reduced by the negative mass of its field. (Einstein's theory leads to almost the same, except that there the negative gravitational mass density is twice as large.) One can then write

$$\nabla^2\phi + \frac{1}{2c^2}(\nabla\phi)^2 = 4\pi\gamma\rho\,. \qquad (22)$$

The earth is therefore embedded in a sea of negative mass. Making the substitution

$$\psi = e^{\phi/c^2} \qquad (23)$$

transforms (22) into

$$\nabla^2\psi = \frac{4\pi\gamma\rho}{c^2}\psi\,. \qquad (24)$$

If ρ is a delta function at *r* = 0, where

$$m = \int 4\pi r^2 \delta(r)\rho\, dr\,, \qquad (25)$$

then one obtains from (24):

$$\frac{d}{dr}\left(r^2\frac{d\psi}{dr}\right) = \frac{m\gamma\psi}{c^2} \qquad (26)$$

or $\phi = \dfrac{m\gamma}{r}$, (27)

the same as in Newton's theory.

For a sphere of constant density ρ one obtains the solution

$$\psi = \frac{\sinh kr}{kr}, \; k^2 = \frac{4\pi\gamma\rho}{c^2}. \tag{28}$$

Hence,

$$\phi = c^2 \ln\left[\frac{\sinh kr}{kr}\right]. \tag{29}$$

For small values of *r* one has

$$\phi = (2\pi/3)\gamma\rho r^2 \tag{30}$$

with the force per unit mass

$$|\mathbf{g}| = -|\nabla\phi| = (4\pi/3)\gamma\rho r, \tag{31}$$

as in Newton's theory.

In general one has

$$|\mathbf{g}| = c^2\left[\frac{1}{r} - \frac{\sqrt{4\pi\gamma\rho}}{c} ctnh\left(\frac{\sqrt{4\pi\gamma\rho}}{c} r\right)\right], \tag{32}$$

which in the limit $r \to \infty$ becomes a constant:

$$|\mathbf{g}| = -\sqrt{4\pi\gamma\rho}c, \tag{33}$$

which implies the self-shielding of a large mass by the negative mass of its own gravitational field surrounding the mass. The shielding becomes important at the distance

$$R = c/\sqrt{4\pi\gamma\rho} \,. \tag{34}$$

Inserting into (34) the average mass density of the universe, *R* becomes the radius of the universe. The negative gravitational mass of the universe there shields its positive mass.

4. The Theory of Bondi Revisited

We are now in a position to revisit the theory by Bondi. In his treatment of the positive-negative-mass dipole two body problem, he did not take into account the gravitational field of this configuration. According to (27) the gravitational potential of a point mass remains the same as in Newton's theory. This means that the gravitational potential interaction energy for two positive equal masses,

$$E_{pot} = -\gamma \frac{m \times m}{r} = -\frac{\gamma m^2}{r} \tag{35}$$

is changed for a mass dipole into

$$E_{pot} = -\gamma \frac{m \times (-m)}{r} = \frac{\gamma m^2}{r} \,. \tag{36}$$

In the theory of Bondi this positive field mass must be added to the positive mass, resulting in a mass pole-dipole, which is a mass pole with a superimposed mass dipole. As we will see in the next section this fact has very important consequences [4].

5. The Zitterbewegung Phenomenon as a Manifestation of Negative Masses

There is no fundamental physical principle standing in the way which forbids the existence of negative masses. If this is true, the question is: Where are these negative masses? The recently noticed large bubbles or voids observed in intergalactic space could possibly be explained by the repulsive force of negative masses assumed to occupy the voids, but alternative, less exotic, explanations have been offered as well. However, there is at least one fundamental phenomenon which strongly speaks for the existence of negative masses. It is the spin of the fermions, like the spin of the electron. Fermions are described by Dirac's relativistic wave equation. This equation has both positive and negative energy components and because of the mass-energy relation, it therefore must have negative mass components. According to Schrödinger [4], it is these negative mass components which lead to the phenomenon of the spin. Since the overall mass of the electron is positive, the occurrence of negative masses in the Dirac equation must mean that the electron is a mass pole with a superimposed mass dipole [5].

The spin is definitely not an intrinsic rotational motion of a finite size particle, as older models had suggested it to be. The original model by Uhlenbeck and Goudsmit, for example, cannot possibly be correct because it requires superluminal rotation velocities for an electron with the classical radius $r_0 = e^2/mc^2$.

If we consider the linear motion of a mass dipole (Figure 2), we immediately see that its translation generates angular momentum. Construction of a mass pole with a superimposed mass dipole can simply be done by choosing the positive mass slightly larger than the magnitude of its negative counterpart. For such a pole-dipole particle the center of mass S is outside the line connecting both masses (Figure 3) with its motion taking place on a circle of radius r_c.

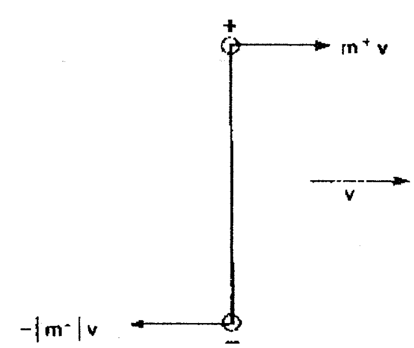

Figure 2. Translation of mass dipole.

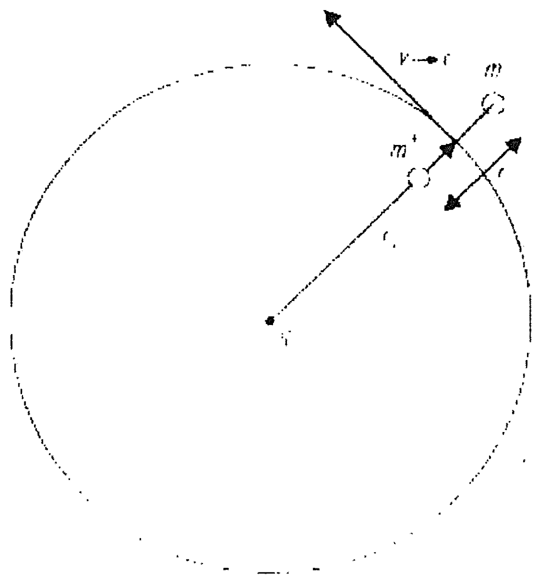

Figure 3. Circular motion of a pole-dipole particle.

Because it is self-accelerating the circular motion will eventually reach the velocity of light. The connection with Schrödinger's analysis is reached if one puts in Planck's constant and m as the electron mass

$$r_c = \frac{h}{2mc}, \tag{37}$$

whereby the circular motion around S produces just the angular momentum $h/2$ as in Dirac's equation.

The velocity-of-light result was first obtained by Breit [6], according to which Dirac's equation predicts a local electron velocity equal to the velocity of light. Even though its time-averaged velocity is always less than the velocity of light, this means that the electron, represented by the pole-dipole configuration, makes a circular luminal motion onto which a subluminal motion of the center of mass S is superimposed. The trajectory of the resulting motion is a screw-line, but it is the motion of S only which one identifies with the (time-averaged) electron velocity. The result derived from this simple pole-dipole model is in beautiful agreement with Schrödinger's analysis of the Dirac equation, in which the luminal rotational motion emerges as a fluctuation of the electron coordinate, called by Schrodinger Zitterbewegung (German for quivering motion).

We may analyze the simple pole-dipole model in more detail as follows. If the positive and negative masses are m^+, m^-, with $m^+ - |m^-| << m^+$, and with the mass pole given by

$$m = m^+ - |m^-|, \tag{38}$$

the mass dipole is

$$p = m^+ r \sim |m^-| r \tag{39}$$

where $r << r_c$ is the distance of separation in between m^+ and m^-. The center of mass is determined by

$$m^- r_c = |m^-| (r_c + r). \tag{40}$$

The angular momentum then becomes (ω is the angular velocity of circular motion with $r_c \omega \to c$):

$$|J| = \left| \left[m^+ r_c^2 - |m^-| (r_c + r)^2 \right] \omega \right|$$

$$\begin{aligned} &= m^+ r_c r \omega \\ &= m^+ r c \end{aligned} \tag{41}$$

Comparing (40) with (37) shows that

$$2m^+ rc = \mathrm{h} \tag{42}$$

and

$$m/m^+ = r/r_c. \tag{43}$$

Experimentally, the electron is indistinguishable from a point. This would make $r = 0$. In reality its size must be finite but in principle can be very small. This means that m^+ (and $|m^-|$) are likely to be very much larger than m.

t has been conjectured by Hönl and Papapetrou [5] that the electron is a pole-dipole particle where the surplus positive energy comes from the positive gravitational interaction energy of a very large positive m^+ mass with a likewise very large negative $m^- = -|m^+|$ mass. According to this hypothesis one would have for the electron rest mass energy

$$mc^2 = \frac{\gamma |m^{\prime}|^2}{r}. \tag{44}$$

Combining (44) with (42) one can compute $m^{\prime}$. The result is [5]:

$$m^{\cdot} = \sqrt[3]{\frac{mc\hbar}{2\gamma}} \sim 6 \text{ X } 10^{-13} \text{ g}, \tag{45}$$

larger by a factor 3.6×10^{11} times the mass of the proton.

We therefore see that there are huge amounts of negative masses bound to positive masses in Dirac spinors. It shows that it cannot be a simple matter to free the negative masses from the positive masses. And it explains why the masses of the elementary particles are so much smaller than the Planck mass, $m_p \sim 10^{-5}$ g.

6. Planck Aether Hypothesis [7,8]

We make here the proposition that the fundamental group is SU2, and that by Planck's conjecture the fundamental equations of physics contain as free parameters only the Planck length r_p, the Planck mass m_p and Planck time t_p (γ Newton's constant, $\hbar$ Planck's constant, c the velocity of light):

$$r_p = \sqrt{\frac{h\gamma}{c^3}} \approx 10^{-33} cm$$

$$m_p = \sqrt{\frac{hc}{\gamma}} \approx 10^{-5} g$$

$$t_p = \sqrt{\frac{h\gamma}{c^5}} \approx 10^{-44} s.$$

The assumption that SU2 is the fundamental group means that nature works like a computer with a binary number system. Since SU2 is isomorphic to SO_3, the rotation group in R3, this explains why natural space is three-dimensional.

The Planck's aether conjecture is the assumption that the vacuum of space is densely filled with an equal number of positive and negative Planck mass particles, with each Planck length volume on the average occupied by one Planck mass, with the Planck mass particles interacting with each other by the Planck force over a Planck length, and with Planck mass particles of equal sign repelling and those of opposite sign attracting each other. The particular choice made for the sign of the Planck force is the only one that keeps the Planck aether stable. While Newton's action-reaction remains valid for the interaction of equal Planck mass particles, it is violated for the interaction of a positive with a negative Planck mass particle, even though globally the total linear momentum of the Planck mass plasma is conserved, with the recoil absorbed by the Planck aether as a whole.

It is the local violation of Newton's action-reaction which leads to quantum mechanics at the most fundamental level, as can be seen as follows: Under the Planck force $F_p = m_p c^2 / r_p$, the velocity fluctuation of a Planck mass particle interacting with a Planck mass particle of opposite sign is $\Delta v = \left(F_p/m_p\right)t_p = \left(c^2/r_p\right)\left(r_p/c\right) = c$, and hence yields the momentum fluctuation $\Delta p = m_p c$. But since $\Delta q = r_p$ and because $m_p r_p c = h$, one obtains Heisenberg's uncertainty relation $\Delta p \Delta q = h$ for a Planck-mass particle. Accordingly, the quantum fluctuations are explained by the interaction with hidden negative masses, with energy borrowed from the sea of hidden negative masses.

According to Newtonian mechanics and Planck's conjecture, the interaction of a positive with a negative Planck-mass particle leads to a velocity fluctuation δ& $= a_p t_p = c$ with a displacement of the particle equal to $\delta = (1/2) a_p t_p^2 = r_p / 2$, where $a_p = F_p / m_p$. Therefore, a Planck-mass particle immersed in the Planck aether makes a stochastic quivering motion (Zitterbewegung) with the velocity

$$v_D = -(r_p c/2)(\nabla n/n), \tag{46}$$

where $n = 1/r_p^3$ is the average number density of positive or negative Planck mass particles. The kinetic energy of this diffusion process is given by

$$\left(\frac{m_p}{2}\right) v_D^2 = \left(\frac{m_p}{8}\right) r_p^2 c^2 \left(\frac{\nabla n}{n}\right)^2 = \left(\frac{\mathrm{h}^2}{8m_p}\right)\left(\frac{\nabla n}{n}\right)^2 . \tag{47}$$

Putting

$$\mathbf{v} = \frac{\mathrm{h}}{m_p} \nabla S , \tag{48}$$

where S is the Hamilton action function and $\boldsymbol{v}$ is the velocity of the Planck aether, the Lagrange density for the Planck aether is

$$L = n\left[\mathrm{h}\frac{\partial S}{\partial t} + \frac{\mathrm{h}^2}{2m_p}\left(\nabla S^2\right) + U + \frac{\mathrm{h}^2}{8m_p}\left(\frac{\nabla n}{n}\right)^2\right]. \tag{49}$$

Variation of (49) with regard to S according to

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial S/\partial t}\right) + \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial S/\partial r}\right) = 0 \tag{50}$$

leads to

$$\frac{\partial n}{\partial t} + \frac{\mathrm{h}}{m_p}\nabla(n\nabla S) = 0 \tag{51}$$

or

$$\frac{\partial n}{\partial t} + \nabla(n\mathbf{v}) = 0\,, \tag{52}$$

which is the continuity equation of the Planck aether. Variation with regard to *n* according to

$$\left(\frac{\partial L}{\partial n}\right) - \frac{\partial}{\partial r}\left(\frac{\partial L}{\partial n/\partial r}\right) = 0 \tag{53}$$

leads to

$$\mathrm{h}\frac{\partial S}{\partial t} + U + \frac{\mathrm{h}^2}{2m_p}\left(\nabla S^2\right) + \frac{\mathrm{h}^2}{4m_p}\left[\frac{1}{2}\left(\frac{\nabla n}{n}\right)^2 - \frac{\nabla^2 n}{n}\right] = 0 \tag{54}$$

or

$$\mathrm{h}\frac{\partial S}{\partial t} + U + \frac{\mathrm{h}^2}{2m_p}\left(\nabla S^2\right) + \frac{\mathrm{h}^2}{2m_p}\frac{\nabla^2\sqrt{n}}{\sqrt{n}} = 0\,. \tag{55}$$

With the Madelung transformation

$$\psi = \sqrt{n}e^{iS}\,,\ \ \psi^* = \sqrt{n}e^{-iS}\,, \tag{56}$$

one obtains from (51) and (55) the Schrödinger equation

$$i\mathrm{h}\frac{\partial \psi}{\partial t} = -\frac{\mathrm{h}^2}{2m_p}\nabla^2\psi + U\psi\,. \tag{57}$$

In the Planck aether hypothesis all particles, save and except the Planck mass particles, are quasi-particles of the Planck aether, like the phonons, rotons, excitons, etc., of condensed matter physics, and by the wave structure of the Planck aether are Lorentz invariant. In forming quantized vortices, the Planck aether also has vortex waves, simulating Maxwell's and Einstein's electromagnetic and gravitational waves. Dirac spinors are made possible by the negative masses of the Planck aether.

Quantum mechanics predicts for each harmonic oscillator the zero-point energy $(1/2)\hbar\omega$, which has to be multiplied with the volume element in frequency space $4\pi\omega^2 d\omega$, to obtain the zero-point energy spectrum

$$f(\omega)d\omega = \text{const} \times \omega^3 d\omega. \tag{58}$$

Now (58) turns out to be just the only spectrum that is Lorentz invariant. But the spectrum (58) is also the only one which does not lead to a friction force on a charged particle moving through an electromagnetic spectrum with this frequency dependence. This means that special relativity is a consequence of quantum mechanics, leading to the zero point vacuum energy, and can be interpreted by saying that the zero-point vacuum energy "generates" the Minkowski space-time.

7. Dynamic Interpretation of Lorentz Invariance

A cut-off at the Planck frequency generates a distinguished reference system in which the zero-point energy spectrum is isotropic and at rest. In this distinguished reference system, the scalar potential from which the forces are to be derived satisfies the inhomogeneous wave equation:

$$-\frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} + \nabla^2\Phi = -4\pi\rho(\mathbf{r},t)\,, \tag{59}$$

where $\rho(\mathbf{r},t)$ are the sources of this field. For a body in static equilibrium at rest in the distinguished reference system for which the sources are those of the body itself one has

$$\nabla^2\Phi = -4\pi\rho(\mathbf{r})\,. \tag{60}$$

If set into absolute motion with the velocity v along x-axis, the coordinates of the reference system at rest with the moving body are obtained by the Galilei transformation:

$$\begin{aligned} x' &= x - vt, \\ y' &= y, \\ z' &= z, \\ t' &= t \end{aligned} \tag{61}$$

transforming (59) into

$$-\frac{1}{c^2}\frac{\partial^2\Phi'}{\partial t'^2}+\frac{2v}{c^2}\frac{\partial^2\Phi'}{\partial x'\partial t'}+\left(1-\frac{v^2}{c^2}\right)\frac{\partial^2\Phi'}{\partial x'^2}+\frac{\partial^2\Phi'}{\partial y'^2}+\frac{\partial^2\Phi'}{\partial z'^2}=-4\pi\rho'(\mathbf{r}',t'). \tag{62}$$

After the body has settled into a new equilibrium in which $\partial/\partial t' = 0$, one has instead of (60)

$$\left(1-\frac{v^2}{c^2}\right)\frac{\partial^2\Phi'}{\partial x'^2}+\frac{\partial^2\Phi'}{\partial y^2}+\frac{\partial^2\Phi'}{\partial z^2}=-4\pi\rho(x',y,z). \tag{63}$$

Comparison of (63) with (60) shows that the left-hand side of (63) is the same if one sets $\Phi' = \Phi$ and $dx' = dx\sqrt{1-v^2/c^2}$. This implies a uniform contraction of the body by the factor $\sqrt{1-v^2/c^2}$ because the sources are contracted by the factor $\sqrt{1-v^2/c^2}$ as well, whereby the right-hand side of (63) becomes equal to the right-hand side of (60). Since the zero-point energy is invariant under a Lorentz transformation, the quantum potential changes in the same way as Φ. The body therefore sustains its static equilibrium under a contraction by the factor $\sqrt{1-v^2/c^2}$ if set into absolute motion, explaining the Lorentz contraction dynamically.

The clock retardation effect can be derived from the contraction effect, and from there the Lorentz transformation. Following Builder [9] this original interpretation of Lorentz invariance by Lorentz and Poincaré has been worked out in every detail by Prokhovnik [10]. To derive the clock retardation effect from the contraction effect one considers a light clock, which is a rod with mirrors attached to its two ends in between which a light signal is sent forth and back. If the length of the rod is *l*, and if the rod rests in the distinguished reference system, the time needed for the light signal to be sent forth and back is

$$t_0 = 2l/c\ . \tag{64}$$

If prior to being set into motion the rod is inclined against the x-axis by the angle φ, it appears to be inclined against the x-axis by the different angle ψ after set into motion, with ψ expressed through φ by

$$\tan\psi = \gamma \tan\varphi \tag{65}$$

$$\gamma = \left(1 - v^2/c^2\right)^{-1/2}.$$

The absolute motion then contracts the rod from l to l':

$$l' = l\sqrt{1-\left(v^2/c^2\right)\cos^2\varphi} = \frac{l}{\gamma\sqrt{1-\left(v^2/c^2\right)\sin^2\psi}}\ . \tag{66}$$

Relative to the moving rod the velocity of light is anisotropic, and for the to and fro directions given by

$$\begin{aligned} c_- &= \sqrt{c^2 - v^2\sin^2\psi} - v\cos\psi \\ c_- &= \sqrt{c^2 - v^2\sin^2\psi} + v\cos\psi \end{aligned} \tag{67}$$

with the time t' for a to and fro signal given by

$$t' = l'/c_+ + l'/c_- = \gamma t_0\ . \tag{68}$$

Therefore, as seen from an observer at rest in the distinguished reference system the clock goes slower by the factor $\gamma = 1/\sqrt{1-v^2/c^2}$, independent of the inclination of the rod making up the clock. With solid bodies held together by electromagnetic forces, clocks made from solid matter should behave like light clocks. As it was claimed by Poincaré, it should for this reason be possible to obtain the Lorentz transformations solely from the contraction effect with a proper convention about the synchronization of clocks.

According to Einstein, two clocks, A and B, are synchronized if

$$t_B = \frac{1}{2}\left(t_A^1 + t_A^2\right), \tag{69}$$

where t_A^1 is the time a light signal is emitted from A to B, reflected at B back to A, arriving at A at the time t_A^2, and where it is assumed that the time t_B at which the reflection at B takes place is equal to the arithmetic average of t_A^1 and t_A^2. Only by making this assumption does the velocity of light turn out always to be isotropic and equal to *c*. From an absolute point of view, the following rather is true. If t_R is the absolute reflection time of the light signal at clock B, one has for the out and return journeys of the light signal from A to B and back to A, if measured by an observer in an absolute system at rest in the distinguished reference system:

$$\begin{aligned} \gamma(t_R - t_A^1) &= d/c \\ \gamma(t_A^2 - t_R) &= d/c \end{aligned}, \tag{70}$$

where *d* is the distance between both clocks, and where c_+ and c_- are given by (67). Adding the equations (70) one obtains

$$c(t_A^2 - t_A^1) = 2\gamma d\sqrt{1-\left(v^2/c^2\right)\sin^2\psi}. \tag{71}$$

If an observer at rest with the clock wants to measure the distance from A to B, he can measure the time it takes a light signal to go from A to B and back to A. If he assumes that the velocity of the light is constant and isotropic in all inertial reference systems, including the one he is in, moving together with A and B with the absolute velocity *v*, the distance is

$$d' = (c/2)(t_A^2 - t_A^1). \tag{72}$$

And because of (71)

$$d' = \gamma d\sqrt{1-\left(v^2/c^2\right)\sin^2\psi}. \tag{73}$$

Comparing this result with (66), one sees that he would obtain the same distance d', if he uses a contracted rod as a measuring stick, or Einsteins's constant light velocity postulate. The velocity of light between A and B by using a rod to measure the distance and the time it takes a light signal in going from A to B and back to A, of course, will turn out to be equal to *c*, because according to (72)

$$\frac{2d'}{t_A^2 - t_A^1} = c \,. \tag{74}$$

Rather than using a reflected light signal to measure the distance d', the observer at A may try to measure the one-way velocity of light by first synchronizing the clock B with A and then measure the time for a light signal to go from A to B. However, since this synchronization procedure also uses reflected light signals, the result is the same. For the velocity he finds

$$\frac{d'}{t_B - t_A^1} = \frac{d'}{1/2(t_A^1 + t_A^2) - t_A^1} = \frac{2d'}{t_A^2 - t_A^1} = c \,. \tag{75}$$

By subtracting the equations (70) one finds that

$$t_R = t_B + \left(\gamma/c^2\right) vd \cos\psi \,, \tag{76}$$

which shows that from an absolute point of view the "true" reflection time t_R at clock B is only then equal to t_B, if $v = 0$. From an absolute point of view the propagation of light is isotropic only in the distinguished reference system, but anisotropic in a reference system in absolute motion against the distinguished reference system. This anisotropy remains hidden due to the impossibility to measure the one-way velocity of light. The impossibility is expressed in the Lorentz transformations themselves, containing the scalar c^2 rather than the vector $\mathbf{c}$, through which an anisotropic light propagation would have to be expressed.

8. Negative Mass Interpretation of the Aharonov-Bohm Effect [11]

In Maxwell's equations the electric and magnetic fields can be expressed through a scalar potential Φ and a vector potential $\boldsymbol{A}$:

$$\begin{aligned} \mathbf{E} &= -\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} - grad\Phi \\ \mathbf{H} &= curl\mathbf{A} \end{aligned} \quad (77)$$

E and **H** remain unchanged under the gauge transformation of the potentials

$$\begin{aligned} \Phi' &= \Phi - \frac{1}{c}\frac{\partial f}{\partial t} \\ \mathbf{A}' &= \mathbf{A} + \mathrm{grad}\, f \end{aligned} \quad (78)$$

where f is called the gauge function. Imposing on Φ and A the Lorentz gauge condition,

$$\frac{1}{c}\frac{\partial \Phi}{\partial t} + div\mathbf{A} = 0, \quad (79)$$

the gauge function must satisfy the wave equation

$$-\frac{1}{c^2}\frac{\partial^2 f}{\partial t^2} + \nabla^2 f = 0. \quad (80)$$

In an electromagnetic field the force on a charge e is

$$F = e\left[\mathbf{E} + \frac{1}{c}\mathbf{v}\times\mathbf{H}\right]$$

$$= e\left[-\frac{1}{c}\frac{\partial \mathbf{A}}{\partial t} - grad\Phi + \frac{1}{c}\mathbf{v}\times curl\mathbf{A}\right]. \quad (81)$$

By making a gauge transformation of the Hamilton operator in the Schrödinger wave equation, the wave function transforms as

$$\psi' = \psi \exp\left[\frac{ie}{\mathrm{h}c} f\right], \tag{82}$$

leaving invariant the probability density $\psi^*\psi$.

To give gauge invariance a hydrodynamic interpretation, we compare (81) with the force acting on a test body of mass m placed into the moving Planck aether. This force follows Euler's equation and is

$$F = m\frac{dv}{dt} = m\left[\frac{\partial v}{\partial t} + grad\left(\frac{v^2}{c}\right) - \mathbf{v}\times curl\,\mathbf{v}\right]. \tag{83}$$

Complete analogy between (81) and (83) is established if one sets

$$\begin{aligned} \mathbf{\Phi} &= -\frac{m}{2e}v^2 \\ \mathbf{A} &= -\frac{mc}{e}\mathbf{v} \end{aligned} \tag{84}$$

According to (78) and (82), Φ and *A* shift the phase of a Schrödinger wave by

$$\delta\varphi = \frac{e}{\mathrm{h}}\int_{t_1}^{t_2} \Phi\, dt$$

$$\boldsymbol{\delta\varphi} = -\frac{e}{\hbar c}\oint \mathbf{A}\cdot \boldsymbol{ds}. \tag{85}$$

The corresponding expressions for a gravitational field can be directly obtained from the equivalence principle [3]. If $\partial\mathbf{v}/\partial t$ is the acceleration and ω the angular velocity of the universe relative to a reference system assumed to be a rest, the inertial forces in this system are

$$F = m\left[\frac{\partial\mathbf{v}}{\partial t} + \dot{\boldsymbol{\omega}}\times\mathbf{r} - \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r}) - \dot{\mathbf{r}}\times 2\boldsymbol{\omega}\right]. \tag{86}$$

For (86) we write

$$F = m\left[\hat{\mathbf{E}} + \frac{1}{c}\mathbf{v}\times\hat{\mathbf{H}}\right], \qquad (87)$$

where

$$\hat{\mathbf{E}} = \frac{\partial v}{\partial t} + \dot{\boldsymbol{\omega}}\times\mathbf{r} - \boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r})$$

$$\hat{\mathbf{H}} = -2c\boldsymbol{\omega}. \qquad (88)$$

With

$$curl(\dot{\boldsymbol{\omega}}\times\mathbf{r}) = 2\dot{\boldsymbol{\omega}}$$

$$div(-\boldsymbol{\omega}\times(\boldsymbol{\omega}\times\mathbf{r})) = 2\omega^2, \qquad (89)$$

one has

$$div\hat{\mathbf{H}} = 0$$

$$\frac{1}{c}\frac{\partial\hat{\mathbf{H}}}{\partial t} + curl\hat{\mathbf{E}} = 0. \qquad (90)$$

$\hat{\mathbf{E}}$ and $\hat{\mathbf{H}}$ can be derived from a scalar and vector potential

$$\hat{\mathbf{E}} = -\frac{1}{c}\frac{\partial\hat{\mathbf{A}}}{\partial t} - grad\Phi$$

$$\hat{\mathbf{H}} = curl\hat{\mathbf{A}}. \qquad (91)$$

Applied to a rotating reference system one has

$$\hat{\Phi} = -\frac{1}{2}(\boldsymbol{\omega}\times\mathbf{r})^2$$
$$\hat{\mathbf{A}} = -c(\boldsymbol{\omega}\times\mathbf{r}) \qquad (92)$$

or

$$\hat{\Phi} = -\frac{v^2}{2}$$
$$\hat{\mathbf{A}} = -c\mathbf{v} \qquad (93)$$

Apart from the factor *m/e*, this is same as (84).

For weak gravitational fields produced by slowly moving matter, Einstein's linearized gravitational field equations permit the gauge condition (replacing the Lorentz gauge)

$$\frac{4}{c}\frac{\partial\Phi}{\partial t} + div\hat{\mathbf{A}} = 0$$

$$\frac{\partial\hat{\mathbf{A}}}{\partial t} = 0\,.$$

With the gauge transformation for Φ and *A*

$$\Phi' = \Phi$$
$$\hat{\mathbf{A}}' = \hat{\mathbf{A}} + \mathrm{grad}\, f \qquad (94)$$

where f has to satisfy the potential equation

$$\nabla^2 f = 0\,. \qquad (95)$$

For a stationary gravitational field the vector potential changes phase of the Schrödinger wave function according to

$$\psi' = \psi \exp\left[\frac{im}{\mathrm{h}c} f\right], \qquad (96)$$

leading to phase shift on a closed path

$$\delta\varphi = -\frac{m}{\hbar c}\oint \overset{\wedge}{\mathbf{A}} \cdot d\mathbf{s}$$

$$\delta\varphi = -\frac{m}{\hbar}\oint \mathbf{v} \cdot d\mathbf{s} \qquad (97)$$

In the hydrodynamic interpretation suggested by the Planck aether hypothesis, the phase shifts caused by either the magnetic or the gravitational vector potential result from a circular flow of the Planck aether. The principle of equivalence can precisely relate this circular flow to the angular velocity of a rotating platform. According to (93), one has for the gravitational vector potential in a rotating frame of reference

$$\hat{\mathbf{A}} = -\boldsymbol{\omega} c r$$

with the phase shift given by (97). One can apply (97) to the Sagnac effect for photons of frequency ν. By putting $mc^2 = h\upsilon = 2\pi\hbar\upsilon$, with the result that

$$\delta\varphi = -\frac{2\pi v}{c^2}\oint \mathbf{v} \cdot d\mathbf{s}$$

$$\delta\varphi = 2\omega\pi r^2 \frac{2\pi v}{c^2} \qquad (97)$$

the same as predicted by quantum mechanics.

We now compute the phase shift (85) by a magnetic vector potential. To make a comparison with the gravitational vector potential in the Sagnac effect, we consider the magnetic field produced by an infinitely long cylindrical solenoid of radius *R*. Inside the solenoid the field is constant, vanishing outside. If the magnetic field inside the solenoid is *H*, the vector potential is

$$A_\varphi = \frac{1}{2}Hr \qquad r < R$$

$$A_\varphi = \frac{1}{2}\frac{HR^2}{r} \qquad r > R. \qquad (98)$$

According to (85) the vector potential on a closed path leads to the phase shift

$$\delta\varphi = -\frac{e}{\hbar c} H \pi r^2 \qquad r < R$$

$$\delta\varphi = -\frac{e}{\hbar c} H \pi R^2 \qquad r > R. \qquad (99)$$

As noted by Aharonov and Bohm [11], there is a phase shift for $r > R$, even though for $r > R$, $H = 0$ (because for $r > R$, curl $\boldsymbol{A} = 0$).

Expressing $\boldsymbol{A}$ by (84) through $\boldsymbol{v}$, the hypothetical circular aether velocity, one has

$$v_\varphi = -\frac{e}{2mc} Hr \qquad r < R$$

$$v_\varphi = -\frac{e}{2mc} \frac{HR^2}{r} \qquad r > R. \qquad (100)$$

One sees that inside the coil the velocity profile is the same as in a rotating frame of reference, having outside the coil the form of potential vortex. If expressed in terms of the aether velocity the phase shift becomes

$$\boldsymbol{\delta\varphi} = -\frac{m}{\hbar} \oint \mathbf{v} \cdot \boldsymbol{ds} \qquad (101)$$

the same as (97) for the vector potential created by a gravitational field, and hence the same as in the Sagnac experiment and neutron interference experiment. But for the magnetic vector potential the aether velocity can easily become much larger than in any rotating platform experiment. According to (102), the velocity reaches a maximum at $r = R$, where it is

$$\frac{|v_{max}|}{c} = \frac{eHR}{2mc^2}. \qquad (102)$$

For electrons this is $|v_{max}|/c = 3\times10^{-4} HR$, where H is measured in Gauss. For $H = 10^4$ G, this would mean that $v_{max} \geq c$ for $R > 0.3$ cm. If this would be same aether velocity felt on a rotating platform, it would lead to an enormous centrifugal and Coriolis field inside the coil, obviously not observed.

The Planck aether model can give a simple explanation for this paradox. The Planck aether consists of two superfluid components, one composed of positive Planck masses and the other one of negative Planck masses. The two components can freely flow through each other, making possible two configurations, one where both components are corotating and one where they are counterrotating. The corotating configuration is realized on a rotating platform, where it leads to the Sagnac and neutron interference effects. This suggests that in the presence of the magnetic vector potential the two superfluid components are counterrotating. Outside the coil, where curl $\boldsymbol{A} = 0$, the magnetic energy density vanishes, implying that the magnitudes of both velocities are exactly the same. Inside the coil, where curl $\boldsymbol{A} \neq 0$, there must be a small imbalance in the velocity of the positive over the negative Planck masses to result in a positive energy density.

9. Negative Masses in Cosmology

In the Planck aether theory, the negative gravitational field energy surrounding a mass is due to an excess of negative over positive mass, and the negative mass surrounding a highly collapsed spherical body or black hole is of the same order of magnitude as the positive mass accumulated inside the collapsed body. An assembly of interacting positive and negative masses can, in general, not lead to a thermodynamic equilibrium. If all the positive masses are separated from the negative ones, it is sufficient to require that each mass species reaches thermodynamic equilibrium. It was shown by Vysin [12] that an assembly of negative masses can acquire thermodynamic equilibrium provided the temperature is negative. The kinetic energy of a negative Planck mass is negative, and an assembly of negative Planck masses has, for this reason, a negative temperature. It therefore can reach thermodynamic equilibrium.

We now make the following hypothesis:

> *If an assembly of positive and negative masses, with the total energy equal to zero, is brought together, the temperature and hence entropy of the mixture will go to zero.*

This hypothesis is the only one consistent with Nernst's theorem. It, of course, implies that an assembly of positive and negative masses can perfectly mix because otherwise no equilibrium can be reached. To satisfy this hypothesis, we assume that the negative masses have negative entropy. Only the assumption that an assembly of negative masses has negative entropy permits an analytic continuation of the entropy from positive to negative temperatures. If the entropy for negative temperatures would be counted positive, the function dS/dT would be discontinuous at $T = 0$.

For the entropy of a mixture of positive and negative masses to become zero requires an exact correlation in the disorder of the positive mass gas with the disorder of the negative mass gas. This is certainly true if the negative mass is equal to the negative mass of the gravitational field of the positive mass, because the Newtonian gravitational field of each particle, all the way down to the smallest dimension, is precisely correlated to the position of the particle. The entropy of the positive mass of matter and the entropy of the negative mass of its gravitational field (correlated to the entropy of the positive mass which is the source of this field) might therefore be called complementary (like a positive and negative photographic image). The expansion from a very small phase space volume would then be possible, because if the positive and negative masses are densely packed within the same volume, not only their energy, but also their entropy would cancel.

The time needed to bring back the universe to its original low entropy state, is the Poincaré recurrence time. While under normal conditions this time is huge, it may in a dense mixture of positive and negative masses with a divergent acceleration become quite small. This might explain why the initial entropy of the universe is very small. The Planck aether hypothesis gives a plausible explanation for the observed vanishing of the sum of all charges, like the electric, color and weak charges. With the phenomenon of charge explained to result from the zero point fluctuations of Planck masses bound in vortex filaments and with an equal number of positive and negative Planck masses, the sum of all the charges must vanish. That this should be also be true for the gravitational charges finds its expression in the compensation of the positive energy in the universe by its negative gravitational energy. This compensation explains why the flatness parameter is $\Omega = 1$.

Furthermore, with the sum of all gravitational charges equal to zero, the cosmological constant Λ playing the role of a kind of charge, demands that also $\Lambda = 0$.

Finally, with the negative entropy of the negative Planck masses playing the role of a kind of photographic negative for the positive entropy of the positive Planck masses, the total entropy, made up from the sum of the positive and negative Planck masses, would also be equal to zero.

In summary we may write

$$\sum \text{charges} = 0 \tag{103}$$

with the cosmological consequence

$$\Omega - 1 = \Lambda = S = 0 . \tag{104}$$

The horizon problem is here resolved by superluminal electromagnetic and gravitational shock waves during the high density phase of the cosmological evolution, not by an inflationary expansion of space.

10. The Cusp/Core Problem in Galactic Halos

We have seen that there appears to be strong evidence for the existence of negative matter in the universe. And we have also given reasons that negative matter might be hidden behind positive matter, forming pole-dipole Dirac spinor configurations neutralizing the negative matter. This still leaves open the question as to whether in certain regions of space there might be a surplus of negative over positive matter, and if negative matter can be "mined" from such regions.

It has been conjectured by Forward [13] that negative matter might be located in the intergalactic voids, explaining the "bubble" structure of the metagalaxy. The negative mass in the voids would produce gravitational potential hills repelling all matter, positive and negative, like the positive matter of the galaxies produces gravitational potential wells attracting all matter, positive and negative. But the accumulation of negative matter in the gravitational wells of positive matter reduces and flattens the depth of the wells. This simple fact might explain the unsolved cusp/core problem of galactic halos [14].

But what happens in the center of galaxies must also happen to a lesser degree in the center of the sun, the planets, the earth and the moon. To "mine" negative matter, if it should exist there, excludes the sun, and also all planets, that have a hot molten core. This does not exclude the moon, however, having the deepest potential well near the earth, with only hot rocks in its center accessible by advanced nuclear mining technology, permitting in principle constructing a tunnel through the moon [15].

11. Searching for Negative Matter in the Gravitational Potential Well of the Moon

The radius of the moon is $R = 1.74 \times 10^8 \text{cm}$ and the gravitational acceleration at its surface is $g_0 \cong 1.62 \times 10^2 \text{cm/s}^2$. At a distance $r < R$ from its center the gravitational acceleration is

$$g(r) = -g_0 (r / R) \qquad (105)$$

and the pressure balance equation is

$$\frac{dp}{dr} = -\rho g_0 \frac{r}{R}, \qquad (106)$$

where $p = p(r)$ is the pressure for $r < R$. The pressure at the center therefore is

$$p_{\max} = \frac{\rho g_0}{R} \int_0^R r dr = \frac{1}{2} \rho g_0 R. \qquad (107)$$

With $\rho \cong 3.33 g / cm^3$ the average density of the moon, one finds that $p_{\max} \cong 5 \times 10^{10} dyn / cm^3 \cong 50{,}000 atm$.

The temperature T can be estimated from the equation $p_{\max} = nkT$, where $k \equiv 1.38 \times 10^{-16} erg/K$ is the Boltzmann constant and $n \cong 10^{23} cm^{-3}$ the atomic number density of the rocks. For $p_{max} \cong 5\times10^{10} dyn/cm^2$ one finds $T \cong 4\times10^3 K$. Both the pressure and the temperature are technically manageable, the pressure with layers of shattered rocks around a tunnel passing through the center of the moon, and the temperature with some cooling. Seismic measurements suggest that the center of the moon is made up of hot rocks.

If appreciable amounts of negative matter have accumulated over billions of years in the center of the moon, it is more likely that this matter is in the form of ultra-light matter, perhaps by an order of magnitude lighter than ordinary matter. There are indications that a Swedish research group has found evidence for the existence of an ultra-dense phase of deuterium, about more than 100,000 times more dense than water [16]. Suppose that in the center of the moon the accumulation of negative matter has led to a form of matter which is 100,000 times lighter than steel, but still has the strength of steel. This would not lead to a negative-positive mass self-chasing mass dipole as envisioned by Forward [13], but to something very important for space-flight, because it would dramatically reduce the energy requirements to accelerate a space craft made from such ultra-light material.

The question as to whether there is such an unusual substance in the center of the moon can probably be answered by seismic wave tomography, obtained by nuclear explosions set off on the surface of the moon.

12. Making a Tunnel through the Moon [15]

The cohesive energy of rocks is of the order $\varepsilon_r \approx 10^{10} erg/cm^3$. Therefore, the explosive yield needed to shatter a spherical volume of radius r is

$$E \cong (4\pi/3)\varepsilon_r r^3. \tag{108}$$

The energy released in a kiloton nuclear explosion is $E \approx 4\times10^{19} erg$. With this energy, the radius of the crushed rocks would be $r \approx 10^3 cm$ (= 10 m), and with a 10 kiloton explosion it would be twice as large.

To make a tunnel, a cylindrical, rather than spherical, volume of crushed rocks is desired. For this reason a thermonuclear shape charge or an explosive lens is better suited to shatter the rocks.

In the center of the moon the temperature is several thousand degrees centigrade. With the heat diffusion equation given by

$$\frac{\partial T}{\partial t} = \chi \nabla^2 T \, , \tag{109}$$

where χ is the heat diffusion coefficient, the diffusion time for a layer of thickness x is

$$\tau = x^2 / \chi \, . \tag{110}$$

For lunar rocks one has $\chi \cong 4\times10^{-3} cm^2/s$. Taking the example $x = 20m = 2\times10^3 cm$, one finds that $\tau = 10^9 s \cong 30 yr$, and at the high rock temperatures the heat diffusion time would be uncomfortably long.

The situation is drastically changed for a layer of crushed rocks, because there it is possible to remove heat by a coolant pumped through the porous medium of the crushed rocks. At the high temperatures of several thousand degrees centigrade, a liquid alkali metal, for example lithium, abundantly available on the moon, could be used as a coolant. The velocity the coolant diffuses into the crushed rocks is determined by Darcy's law

$$v = -D \, \mathrm{grad} \, p \, , \tag{111}$$

where p is the pressure, $D = \kappa / \rho g$, with $\kappa \sim 1$ *cm/s* a typical value. If the pressure gradient is provided by the gravitational force one has $\mathrm{grad} \, p = \rho g$ and hence

$$|v| = \kappa \sim 1 \; cm/s. \tag{112}$$

The time needed for the liquid metal to pass through *a* ~ 20 m thick layer is then ~ 2 × 10^3 seconds ~ 1 hour. The specific heat per unit volume of the coolant is ρc_v ~ 3 x 10^7 *erg/cm³K*, and for $T = 3 \times 10^3$ K one has $\rho c_v T \sim 10^{11}$ *erg/cm³*.

The heat per unit volume which has to be removed from the crushed rocks is of the order *p*, where *p* is the rock pressure. In the center of the moon where $p = 5 \times 10^{10}$ dyn/cm², this energy is 5×10^{10} erg/cm³. It thus follows that the volume of the liquid coolant must be about ½ of the rock volume to be cooled. For a rock volume of (20 cm)³ ~ 10^4 m³, a coolant volume of about 5×10^3 cm³ would be needed. The same coolant can be used many times over after the heat is removed from it, which could be done on the surface of the moon by radiation or perhaps better by heat exchangers transferring the heat to lunar sand. Without a thick layer of shattered rocks surrounding the tunnel, the pressure acting on the tunnel wall would be large, in particular in the center of the moon. Because of friction between particles of the shattered rock, large shear stresses can be sustained changing the pressure distribution in the rock and reducing the pressure gradient and hence the pressure on the tunnel wall.

A more detailed calculation [15] for the pressure distribution in the shattered rock tunnel wall gives

$$p = \left(r/r_0\right)^9, \qquad (113)$$

where r_0 is the radius of the tunnel.

Integrating eqn (108) one obtains for the pressure distribution in the moon

$$p(r) = -\frac{\rho g_0}{R}\int_R^r r\,dr = -\frac{\rho g_0}{R}(R^2 - r^2) \qquad (114)$$

for which one can also write

$$p(r) = p_{max}\left(1 - \left(\frac{r}{R}\right)^2\right). \qquad (115)$$

If r_s is the horizontal radius up to which the rocks at a certain depth have to be shattered (cylindrical case), one finds by equating $p(r)$ in (115) and (117)

$$r_s = r_0 \left({p_{\max}}/{p_0} \right)^{1/9} \left(1 - \left(\frac{r}{R} \right)^2 \right)^{1/9} . \tag{116}$$

The total energy required to shatter the rocks to make a tunnel from the center at the moon where $r = 0$, to its surface where $r = R$, is then given by summing up over the slices with radius r_s and thickness dr

$$E = \pi r_0^2 \varepsilon_r \left(\frac{p_{\max}}{p_0} \right)^{2/9} \int_0^R \left(1 - \left(\frac{r}{R} \right)^2 \right)^{2/9} dr , \tag{117}$$

where as in eqn (110) $\varepsilon_r \cong 10^{10} erg/cm^3$ is the cohesive binding energy of the rocks. For (119) one can write

$$E = \pi r_0^2 R \varepsilon_r \left(p_{\max} / p_0 \right)^{2/9} \int_0^1 \left(1 - x^2 \right)^{2/9} dx . \tag{118}$$

With the help of Euler's betafunction one has

$$\int_0^1 \left(1 - (x)^2 \right)^{2/9} dx = (1/2) B\left(\frac{1}{2}, \frac{11}{9} \right) \cong \frac{\sqrt{\pi}}{2} . \tag{119}$$

Hence,

$$E \cong (\pi^{3/2}/2) r_0^2 R \varepsilon_r \left(p_{\max} / p_0 \right)^{2/9} . \tag{120}$$

Inserting $r_0 = 2 \times 10^3 cm$, $R = 1.74 \times 10^8 cm$, $\varepsilon_r \cong 10^{10} erg/cm^3$, $\left(p_{\max} / p_0 \right) = 5 \times 10^4$, one finds that $E \cong 2 \times 10^{24} erg \cong 5 \times 10^4 kiloton = 50 Megaton.$

It must be emphasized that this energy must be quite nonuniformly released along the tunnel shaft. Nuclear fusion explosions below a yield of 10 kiloton become uneconomical with only a fraction of the energy in the fissionable material (needed to make a critical assembly) released. For a 10 kiloton fission explosion the shatter radius computed from (110) is ~ 20 m. With a tunnel radius r_0 ~ 10 m one would have $(r_s/r_0) \sim 2$ and from (118) that

$$1-\left(\frac{r}{R}\right) \cong 2^9 \left(p_0/p_{max}\right). \qquad (121)$$

Putting for the depth of the tunnel (if measured from the surface of the moon) $\delta = R - r$, with

$$2\delta/R \cong 2^9 \left(p_0/p_{max}\right) \cong 10^{-2} \qquad (122)$$

or that $\delta \cong 10\,km$.

For a depth < 10 km the nuclear explosion with a yield < 10 kiloton would suffice, a yield which is uneconomical. It is for this reason suggested that one uses altogether thermonuclear explosive devices where the cost per yield is much lower. To penetrate and shatter the rocks more efficiently, jet-generating thermonuclear explosive lenses could be used. The thermonuclear detonation wave ignited at one point is there shaped into a jet-producing conical explosion by placing obstacles in the path of the wave. The ignition can be done by a fission explosive, but conceivably also by a powerful laser beam, with the laser beam projected down the tunnel shaft, triggering the thermonuclear explosive positioned at the lower end.

With the above-given estimate of ~ 50 Megaton needed to dig the tunnel shaft, the number of thermonuclear explosive devices making use of the detonation wave lens technique could for this reason be quite reasonable, and certainly much less than the number of required fission explosives.

After nuclear explosions have crushed the rocks and the heat is removed, the tunnel wall has to be made from some kind of ceramic material, since water with which to make concrete is only sparsely available on the moon. But for the wall to last, its temperature must be kept low. The low heat conductivity of rocks, requiring little cooling, is there of considerable help. For the crushed rocks the heat conduction coefficient should not be very different than for solid rocks. According to eqn (112) the heat diffusion time for a 20 m layer of rocks is ~ 30 years. This means that a small, continuous removal of the heat through the injection and circulation of a liquid metal into the crushed rocks should keep down the temperature of the tunnel wall and its environment.

13. Conclusion

The purpose of this study is the question as to whether negative mass might exist, and if negative mass propulsion is possible at all. It is unlikely to be possible in the fashion speculated by Forward [13] (but also see Winterberg [17]), where a negative mass is chasing a positive mass without the expenditure of any energy. Rather, it might perhaps be possible through the existence of an ultra-light form of matter with the tensile strength of ordinary matter on a macroscopic scale where positive matter is bound to negative matter, as happens with Dirac spinor particles on a microscopic scale. This is the speculative existence of macroscopic bodies approaching zero rest mass, of importance for space flight because such matter would greatly reduce its energy requirements.

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